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On sharp spectral lines in the climate record and the millennial peak

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Abstract. The existence of a narrow spectral line in the internal oscillations of the climate system would be a physical discovery of the first order. Thus the appearance of such a spectral peak, with a $Q > 20$ near 1500 years period, in the physical properties of the Greenland ice core is intriguing (bandwidth less than two cycles in 110,000 years). Apparently similar phenomena exist in some deep sea cores. It is shown, however, that the peak appears at precisely the period predicted as a simple alias of the seasonal cycle inadequately (under the Nyquist criterion) sampled at integer multiples of the common year. If this peak is therefore discounted, climate variability appears, as expected, to be a continuum process in the millennial band. More generally, however, aliasing of high frequencies must be a concern in any core analysis.

1. Introduction

A number of recently published papers [*Bond et al.,* 1997; *Mayewski et al.,* 1997; *Grootes and Stuiver,* 1997; *Bianchi and McCave*, 1999] show an apparent spectral peak in ice and ocean cores in the vicinity of 1500 years period, whose presence has led to discussions of the physics of climate change in this so-called millennial band. The peak is remarkably prominent in the Greenland ice core (GISP2) oxygen isotope concentration results of *Grootes and Stuiver* [1997]. Figure 1 shows a spectral reanalysis of the composite Polar Circulation Index (PCI) discussed by *Mayewski et al.* [1997] along with a high-resolution periodogram of the record in the vicinity of the conspicuous peak near 1470 years. Other records, including the deep-sea cores described by *Bond et al.* [1997] and *Bianchi and McCave* [1999], display excess energy in this band, but whether this energy is related to the sharp peak in the ice cores has remained obscure. (Some of the obscurity arises from spectral estimation methods and plotting conventions. The appendix tries briefly to describe some of the problems.)

The periodogram (the raw, squared Fourier coefficients; notice the logarithmic scale) in Figure 1 shows that the peak width does not exceed two cycles in 110,000 years, an extraordinarily sharp periodicity in a record for which sampling-time errors are a major concern. Peak width is best measured in terms of the electrical engineer's "quality factor" [e.g., *Jackson,* 1975],

$$Q = \frac{s_0}{\Delta s} > 20. \quad (1)$$

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Here s_0 is the circular frequency of the peak center, and Δs is defined as the bandwidth of the peak at its half-power points. Even the massive El Niño-Southern Oscillation Index signal in climate has a $Q \sim 1$ or less (an example of its spectrum can be seen in *Wunsch* [1999]). Although there exist published simplified climate system models [e.g., *Macayeal,* 1993; *Sakai and Peltier,* 1995] exhibiting internal narrow-band oscillatory modes, to my knowledge there is no evidence for any pure tones in the climate system other than those produced by external astronomical forcing at the Milankovitch and tidal periods. Simplified systems lack the degrees of freedom that leach energy out of a tuned response through tunneling, modal interactions, and dissipation in real fluids. Confirmation of a pure line component in climate variability, one that is not astronomically forced, would be both astonishing and a major discovery.

2. Explanations

2.1. Aliasing Hypothesis

As the simplest explanation, I suggest that the nearly pure line observed is an alias of the annual cycle present in most of the climate system, and which is usually the dominant variability. "Aliasing" is most familiar as the stroboscope effect, with a high frequency appearing to be a lower one through a failure to sample at a minimum rate of two times per period.

Two different year lengths have major physical significance [e.g., *Thomson,* 1995]: the tropical year of $T_{\rm trop} = 365.2422$ days and the anomalistic year of $T_{\rm anom} = 365.2596$ days. Suppose one has a sinusoid with frequency $s_{\rm trop} = 1/T_{\rm trop}$ of the tropical year and it is sampled at intervals $\Delta t = kT_{\rm com}$ where $T_{\rm com} = 365$

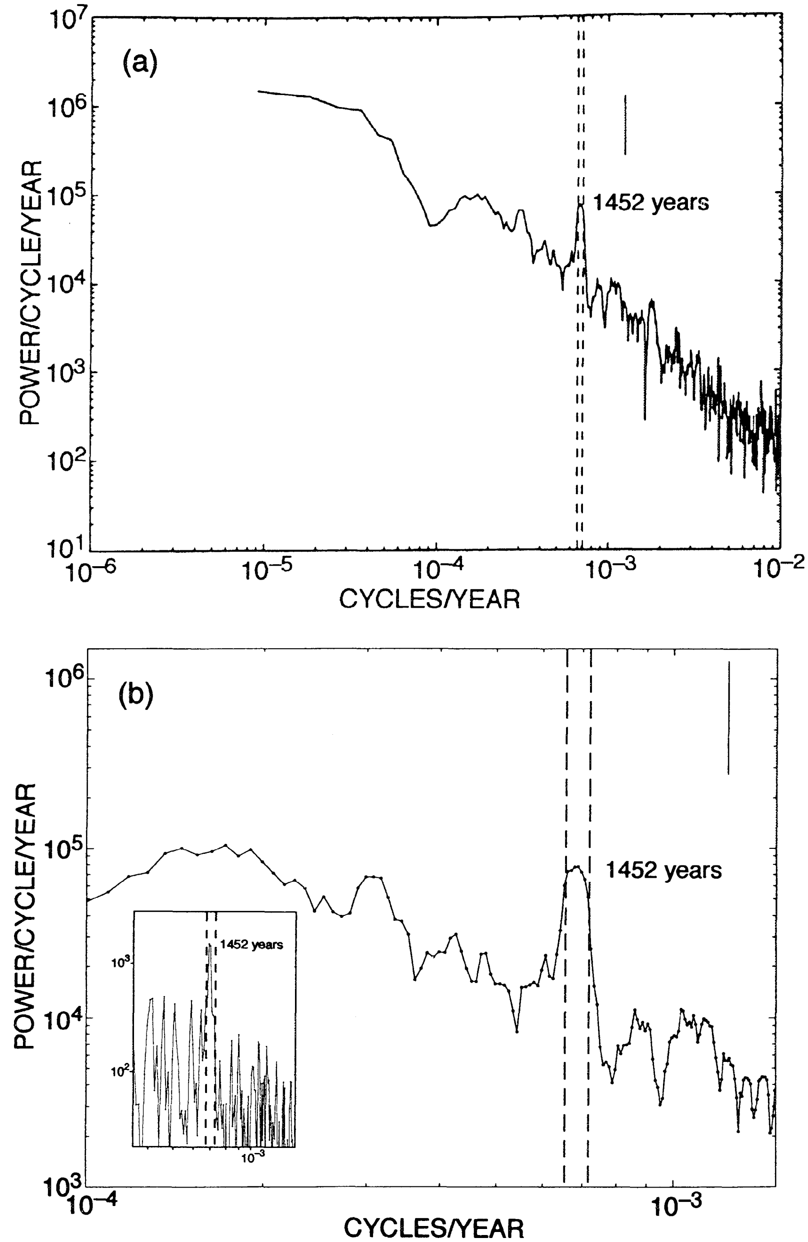

Figure 1. (a) A multitaper spectral estimate of the Polar Circulation Index (PCI) of *Mayewski et al.* [1997]. An approximate mean 95% confidence limit is shown. The peak is a maximum at ~1452 years. The power scale is logarithmic, permitting the use of a near-constant confidence interval shown as a vertical line segment. The vertical dashed lines show the predicted alias position of the anomalistic year at 1/1508 cycles per year and the position of the tropical year at 1/1407 cycles per year. (b)The same as in Figure 1a, except on an expanded scale. The inset is the periodogram (simply the square of the Fourier series coefficients) showing that the peak has a bandwidth of no more than two cycles in 110,000 years. Vertical dashed lines are the same as in Figure 1a.

days is the "common year." Then the apparent, or aliased, frequency of the sampled sinusoid is [*Bracewell,* 1978]

$$s_{\text{alias}} = s_{\text{trop}} - \frac{n}{\Delta t}, \qquad s \geq 0, \tag{2}$$

where n is any integer such that

$$s_{\text{alias}} \leq 1/\left(2\Delta t\right). \tag{3}$$

If $k = 50$ for a sampling interval of 50 common years, only $n = 50$ satisfies (3), and one finds from (2) that the apparent frequency is $s_{\text{alias}} = 1/1508$ years. All integer multiples, $1 \leq k \leq 754$, produce the same alias period. If the calculation is done with $s_{\text{anom}} = 1/T_{\text{anom}}$, the aliased frequency is 1/1407 years. The two aliased periods of 1407 and 1508 years precisely bracket the peak in Figure 1. This result, coupled with the extraordinary sharpness of the peak would, in the words of a reviewer, be "miraculous" if it were mere coincidence. Note that *Mayewski et al.*[1997] interpolated their samples to a nominally uniform interval of 50 years.

One readily confirms that the aliased period is extremely sensitive to the precise value of Δt; if, for example, Δt =365.2422 days and n is an integer satisfying (3), the alias of the tropical year would appear at zero frequency, and the anomalistic year would appear, disturbingly, at 1 cycle/21,000 years. That aliasing is important in understanding core records is not a new idea: *Pisias and Mix* [1988] pointed out the serious problems of interpreting apparent Milankovitch cycles.

Several objections can be raised to this hypothesis of an aliased peak. A summary of the major issues would be as follows:

1. Nature, or the data analyst, filters the physical fields by diffusion, e.g., oxygen isotopes in ice cores [*Johnsen* et al., 1997] or by bioturbation in marine cores [*Goreau,* 1980] or by averaging multiple samples or melting a finite length of ice core, thus reducing the original signal energy. Unless the filtering is perfect, however, some energy always leaks through the filter, and it becomes a quantitative issue as to whether the residual can explain the peak. (The modern seasonal temperature range in Greenland exceeds 30°C.) Note that the peak in Figure 1 contains only ~3% of the record variance; its prominence is an indication of the great power of Fourier methods for the detection of very weak signals in noise.

2. Dating errors would preclude any possibility of a near-uniform sampling interval centered on an integer number of days. The existence of a sharp peak in the spectrum, however, places strong bounds upon the degree and nature of the timing errors. Timing or dating or sedimentation rate errors are an important general issue in discussing cores, and this subject is taken up further below. Experiments (not shown) with a so-called Lomb-Scargle spectral analysis method [*Press et al.,* 1992], one not requiring prior interpolation to a uniform grid and which is less sensitive (but not immune) to aliasing, did not show a significant peak at 1470 years. Unfortunately, this algorithm, which is a fast least squares fit of sinusoids to the data, is sensitive to dating errors owing to potential ill conditioning in the normal equations.

(3) The physical year involves fractional days. How could the sampling interval in the ice core be locked to an integer number of days (365) when the analyst attempts, by counting annual rings [see *Grootes and Stuiver,* 1997, Figure 2] to determine the time of summer? The answer appears to be very simple: It lies with the interpolation not the sampling. An analyst, believing that the small difference between 365 days and 365.2422 days is negligible for a millennial scale signal, chooses the interpolation interval to be an integer multiple of 365 days. But this apparently very slight error is a systematic one, and we have seen (2) and (3) that the alias period of any annual cycle is very sensitive to precisely which interval Δt is chosen. A very small interpolation bias inadvertently produces the sharp aliased peaks.

It is important that most of the energy appearing in the records and that lies visually in the millennial band [e.g., *Mayewski et al.,* 1997, Figure 5], is described by the spectral continuum and not by the narrow aliased peak. When the PCI record is notch-filtered to remove energy between about 1600 and 1300 years periods, the resulting time series is visually very similar (Figure 2) to the original record with the aliased peak present. The major change is in the time interval 3000-4000 years B. P. where some reduction in variance is visible. Two broad bands, one centered near the 21,000 year period and one centered near a 4000 year period, have much of the record energy. Dansgaard/Oeschger/Heinrich events and related phenomena in this range of periods are not described by the spectral peak but by the spectral continuum having a very different physics than a pure tone.

2.2. Alternative Explanations

Solar oscillations are sometimes invoked as a possible source of narrowband exterior forcing. What is known of solar physics (a magnetohydrodynamic system), however, strongly suggests that only broadband oscillations are likely to be present at such periods [*N. Weiss,* 1997; *Tobias et al.,* 1995]. Similarly, the suggestion of the existence of hitherto unappreciated tidal forcing at these periods [*De Rop,* 1971] can be discounted on the quantitative basis [*Doodson,* 1921; *Cartwright and Edden,* 1973] that the forcing amplitude is minuscule.

3. Aliasing as a General Problem

An important point is that aliasing of high frequencies present in cores will always occur, with a magnitude dependent upon the natural or numerical filtering. Having aliased energy appear in a known peak is a desirable outcome relative to the alternative of having it spread over many unknown frequencies, or appearing as it can, in a line at one of the Milankovitch periods [cf. *Pisias and Mix,* 1988].

It may well be true that little or no aliasing contributes to the oscillations seen in Figure 2, but they are not describable as a narrowband process. The degree of aliasing in them would have to be determined from the detailed behavior of the high frequency, undersampled, parts of the core spectrum. If the high frequency spectrum rolls off at least as fast as frequency, s^{-2}, one can alias with impunity [*Wunsch,* 1972]; with a less steep decay with increasing frequency, aliasing will be a se-

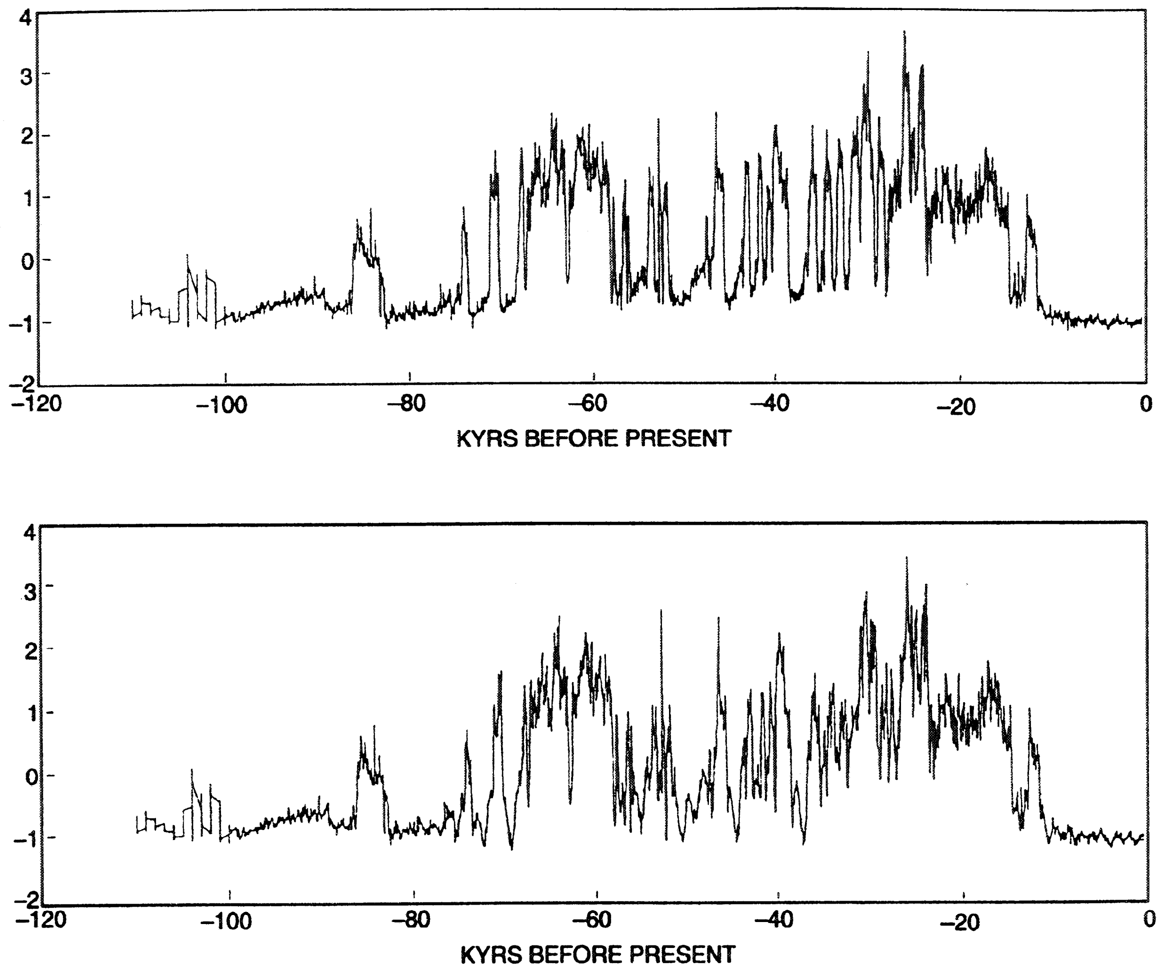

Figure 2. (top) Original PCI index through time. (bottom) The same as in the top plot but with all energy in periods between 1600 and 1300 years removed from the record. Most of the energy visible in the millennial band is untouched by removal of the major spectral peak, which contributes only a small fraction of the total variability.

rious problem, one that cannot be remedied after the fact.

4. Timing (Dating) Errors

Timing errors in cores are inevitable. The size and nature of the errors will vary greatly from core to core and with the nature of the many different physical properties which are measured. Some samples are local averages, and some are point measurements. Consider only one specific issue.

Suppose one takes seriously the suggestion that aliasing of the annual cycle could explain the narrow millennial peak. Is it plausible that the sharp line seen would survive the inevitable "jitter" in the actual sample times? (To the extent that the samples are from local averages, one must also study the effects of filtering, and they are taken up below.) A general theory of this problem has been provided by *Moore and Thomson* [1991] and *Thomson and Robinson* [1996]; here I will display only a simplified calculation, adequate for present purposes.

Consider a unit sine wave

$$y(t) = \sin 2\pi s_0 t. \tag{4}$$

We suppose that $y(t)$ is sampled at times

$$t_j = j\Delta t + \varepsilon_j, \tag{5}$$

where j are integers, Δt is again fixed, and ε_j is a zero mean random timing error. What is the apparent spectrum of $y(t_j)$ if the ε_j are ignored (treated as zero)?

The autocovariance of $y(t_j)$ is defined as

$$R(t_j, t_k) = < y(t_j)\, y(t_k) >, \tag{6}$$

where the brackets denote the theoretical average value. Substituting for y and invoking some basic trigonometric identities, we have

$$
\begin{aligned}
2R(t_j,t_k) = \\
&\cos 2\pi s_0(j-k)\Delta t < \cos 2\pi s_0\left(\varepsilon_j-\varepsilon_k\right) > \\
&-\cos 2\pi s_0(j-k)\Delta t < \sin 2\pi s_0\left(\varepsilon_j-\varepsilon_k\right) > \\
&+\cos 2\pi s_0(j+k)\Delta t < \cos 2\pi s_0\left(\varepsilon_j+\varepsilon_k\right) > \\
&-\cos 2\pi s_0(j+k)\Delta t < \sin 2\pi s_0\left(\varepsilon_j+\varepsilon_k\right) > .
\end{aligned} \tag{7}
$$

Terms of the form $< \sin 2\pi s_0\left(\varepsilon_j \pm \varepsilon_k\right) >$ vanish by symmetry. What is $< \cos 2\pi s_0\left(\varepsilon_j \pm \varepsilon_k\right) >$? If we make the assumption that the timing errors are Gaussian with variance σ^2, then $\xi = \varepsilon_j - \varepsilon_k$ is also a zero mean Gaussian variable with variance $2\sigma^2$, and the value can be evaluated directly as

$$
\begin{aligned}
< &\cos 2\pi s_0 \xi >= \\
&\frac{1}{2\pi^{1/2}\sigma}\int_{-\infty}^{\infty}\cos 2\pi s_0\xi \exp\left(-\xi^2/4\sigma^2\right) \\
= &\exp[-(2\pi s_0\sigma)^2], \qquad j \neq k,
\end{aligned} \tag{8}
$$

independent of j,k. For $j=k$ the value is obviously 1. Otherwise,

$$
\begin{aligned}
&R(t_j,t_k) = \frac{1}{2}\exp\left[-(2\pi s_0\sigma)^2\right] \\
&\cdot\left[\cos 2\pi s_0\left(j-k\right)\Delta t + \cos 2\pi s_0(j+k)\Delta t\right], \\
&j \neq k.
\end{aligned} \tag{9}
$$

Define $\tau = (j-k)\Delta t$. Then

$$
\begin{aligned}
&R(\tau, j) = \frac{1}{2}\exp\left[-(2\pi s_0\sigma)^2\right] \\
&\cdot\left[\cos 2\pi s_0\tau + \cos 2\pi s_0(j+\tau)\Delta t\right], \\
&\tau \neq 0.
\end{aligned} \tag{10}
$$

The Wiener-Khinchin Theorem [e.g., *Percival and Walden*, 1993] states that for a stationary time series the power density spectrum is proportional to the Fourier transform of R over $-\infty \leq \tau \leq \infty$ (including the value for $\tau = 0$). Fixing j and evaluating the Fourier transform produces two contributions, one independent of j and the other directly dependent upon $\cos 2\pi s_0 j$, which when summed over all j, will vanish. The covariance of the mistimed sinusoid is thus

$$
\begin{aligned}
R(\tau) &= 1/2, \qquad \tau = 0 \\
&= \frac{1}{2}\exp\left[-\left(2\pi s_0\sigma\right)^2\right]\cos 2\pi s_0\tau, \qquad \tau \neq 0.
\end{aligned} \tag{11}
$$

For large $s_0\sigma = \sigma/T_0$, $R(\tau)$ approaches a δ function at the origin, which corresponds to a white noise spectrum. For very small $s_0\sigma$, $R(\tau)$ is a pure cosine, whose Fourier transform is a δ function at $\pm s_0$, i.e. a pure line spectrum. For intermediate values of σ/T_0 the spectrum is a pair of δ functions superimposed upon a white noise background such that in all cases the integral over the spectrum is equal to the mean-square value of $y(t_j)$ (Parseval's Theorem). These results are all consistent with the conclusions of *Moore and Thomson* [1991], and the numerical simulations of *Pisias and Mix* [1988]. In summary, a white noise jittered pure sine wave has an apparent power density spectrum which is a sharp peak at the correct frequency, superimposed upon a white noise spectrum.

The results in (11) apply to any frequency s_0 and timing variance σ^2. For the special case of an annual cycle and values of σ less than ~6 months a visible line spectrum survives at the annual period; it would thus alias as described above. With timing error standard deviations exceeding 6 months one sees primarily a white noise spectrum. Lower-frequency sinusoids are correspondingly less attenuated into white noise by these timing errors. If subsampled, the power above the Nyquist frequency, whether it is a line or a white noise continuum, will alias into lower frequencies.

This analysis is clearly a special case. Apart from the Gaussian assumption, any systematic structure in the timing errors (e.g., correlations in time) will modify this result, as will any tendency for the timing errors to vary with depth down a core (as they surely will), rendering the process nonstationary, even should the signal be stationary. Each situation would need to be evaluated individually (see the references already given). Timing errors, however, unless very large, are unlikely to reduce the annual cycle to a pure white noise contribution. Much of the white noise itself will alias, in any case.

The message for deep-sea core dating errors is somewhat encouraging. As long as there are no systematic errors in estimates of dates or sedimentation rates, a sharp peak, e.g., at 42 kyr, will remain a sharp peak, albeit partially attenuated and redistributed into white noise, in the presence of random timing errors of even thousands of years. Presumably this insensitivity is why even "untuned" cores often display surprisingly sharp line spectra at low frequencies, and implying the absence of large systematic timing errors.

5. A Deep-Sea Core

As noted, several papers suggest, on the basis of spectra from deep sea cores, that there are "periodicities" [*Bianchi and McCave,* 1999] in the climate system in this same band. Figure 3 depicts a reanalysis of their record using the multitaper method and plotting the result on a logarithmic scale. There is no evidence for any kind of spectral peak, merely a conventional broad rednoise rise toward ~1800 year period, with a slight decline toward a very low frequency white noise behavior. This record is consistent with the expected broadband character of climate change. Most of the energy in the record lies near periods of 1000 years and produces the

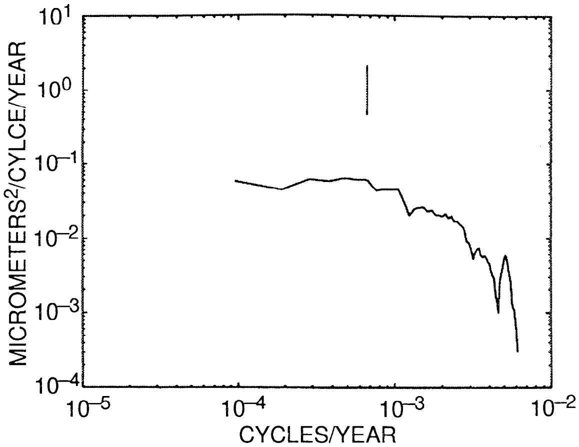

Figure 3. Spectral density estimate of the record discussed by *Bianchi and McCave* [1999]. The record is best described as having a spectral continuum without significant spectral lines. There is no direct evidence of aliasing, although its absence is not assured.

visual character of an oscillatory process in time but without its actually being a periodic one. If the spectral estimates published by *Bianchi and McCave* [1999] are replotted on a logarithmic energy scale (not shown), the same broad-band character is seen: It is not a consequence of the different analysis method but of the linear plotting scale. See the appendix for some comments on spectral plotting.

There is thus evidence neither for a narrowband component in their record nor for any aliasing of the annual cycle although the latter cannot be ruled out. The so-called Dansgaard/Oeschger/Heinrich events, which are supposed to dominate this record on millennial scales, are as expected, spectrally broadband and not periodic.

6. Summary

The conspicuous spectral peak in the millennial band, particularly visible in the Greenland ice cores, appears to be an alias of the annual cycle, as the periods predicted by the aliasing hypothesis so accurately determine its position that a coincidence seems "miraculous." Energy visible in this band in deep-sea cores is dominated not by a narrow spectral line but rather by a broadband continuum consonant with the simplest physical expectations.

No indication exists of any sharp spectral lines in the climate system other than those driven by external astronomical forces, including, conceivably, the mechanism of *Muller and MacDonald* [1995]. The core records are consistent otherwise, with a continuum stochastic process, having varying spectral structure as a function of frequency and containing the great preponderance of the record energy.

Appendix: Comments on Spectral Estimation and Filtering

Some general comments about spectral estimation in the ice/ocean core context may be helpful.

A1. Parametric Methods

Spectral estimation methods based on Fourier analysis are often called "nonparametric" to contrast them with methods such as maximum entropy, allpole, etc.,. which are therefore labeled as "parametric." These latter methods are very powerful because they can build on detailed a priori knowledge of the structure of the spectrum. For example, if one knows that two pure frequencies are present in a time series, that knowledge can be translated into estimates of their amplitudes, even if their frequency separation violates the Rayleigh resolution criterion that says they must differ in frequency by a minimum of one full cycle in the record. Such a determination is known as "superresolution" [see *Munk and Hasselmann*, 1964]. But these procedures are dangerous unless one is prepared to carefully test that the model is the correct one.

Thus the a priori assumption that a narrow line spectrum is present must be carefully examined after the fact. *Percival and Walden* [1993], *Tukey* [1984] and others describe the pitfalls. For the climate system an a priori hypothesis of a sharp line is controversial and its reality should be explicitly demonstrated and not assumed.

A2. Irregular Sampling Methods

Methods exist for estimating the spectra of nonuniformly sampled time series without first interpolating to a uniform grid. One particularly useful form is the so-called Lomb-Scargle algorithm, a fast least squares method, which is summarized by *Press et al.* [1992]. That is to say, it is a periodogram estimate and the only available confidence limits are for the testing of the significance of a pure line frequency in the presence of white noise. The method does have some alias-suppressing capabilities related to the (theoretical) possibility of irrational sampling intervals (see particularly the irregular-sampling theorems discussed by *Freeman* [1965]). In the present context, there are several difficulties: One must determine the significance of peaks in what is clearly a rednoise background and the effects of jitter in the sampling times are likely to be much more severe than for the uniform sample estimators. Further exploitation of this type of analysis is dependent upon an understanding of the spectral confidence limits, and exploration of the effects of timing jitter, exercises which go beyond the scope of what is intended here.

A3. Plotting Spectra

A comment on plotting of spectra may appear gratuitous and in the class of Lord Kelvin's observations on the importance of using a pencil rather than a pen on stripchart recorders. However, the diversity of representations in the paleocommunity does suggest that the reasons for some of the conventions widely used in spectral analysis are not always fully appreciated. A convenient place to begin is with the so-called Parseval relation [*Bracewell*, 1978] which relates the mean-square power in a record x_t to the real Fourier coefficients a_n, b_n,

$$\frac{1}{N}\sum_{t=1}^{N} x_t^2 = \sum_{n=0}^{[N/2]-1} (a_n^2 + b_n^2), \tag{12}$$

assuming no power in the mean or the Nyquist frequency. Note that the Fourier coefficients and the real and imaginary parts of the Fourier transform, evaluated at the Fourier series frequencies ($s_n = n/T, T = N\Delta t$), differ only by a constant multiplier. The left side of (12) is usually defined as the mean-square power in time series x_t. All frequencies s_n are on an equal footing with the frequencies required to fully describe a time series ranging from one cycle per record length, $s = 1/T$, to the Nyquist frequency, $s = 1/2\Delta t$, and their number is equal to the number of data points (half of them in a_n and half in b_n).

Because of the large number of contributors to the Parseval sum it is perfectly possible to have a very large value in one or a few components $a_{n_0}^2 + b_{n_0}^2$, which while visually dominating a power density spectral estimate, contain only a minute fraction of the total energy (see, for example, Figure 3; the spectral density can be regarded as an estimate of $\Phi(s_n) = < a_n^2 + b_n^2 >$, with the brackets denoting the ensemble mean of the values). Use of a logarithmic power scale removes this problem. A second reason for a logarithmic scale is that many geophysical processes have spectra which follow power laws, so that they are proportional in some range to s^{-q}, with q a constant. Such power laws are very powerful theoretical constructs. On a log-log scale the corresponding power densities are linear functions and so easily recognized. Third, power density spectral estimates are themselves stochastic processes and are almost uninterpretable without an estimate of their variance. The variance of any nonparametric estimate $\tilde{\Phi}(s_n)$ is proportional to $\tilde{\Phi}(s_n)^2$ [e.g., *Percival and Walden*, 1993]; that is, the larger the estimated spectral peak, the more uncertain it is. On a logarithmic power scale, however, the confidence interval for $\tilde{\Phi}(s_n)$ plots as a fixed interval independent of $\tilde{\Phi}(s_n)$, rendering it a simple matter to visually estimate peak significance.

A4. Filtering

Deep-sea cores are subject to bioturbation. Because bioturbation can be thought of as a form of low-pass filtering [*Goreau*, 1980], one might conclude that samples from such cores would be immune to the aliasing inferred here. There is, however, no such thing as a perfect filter. Even the most carefully designed numerical filters, acting on long strings of uniformly spaced observations, produce some remnant energy at the original frequency. How much energy is subsequently aliased into a subsampled record depends upon the details of the filter applied and the relative energy of any original pure frequency present.

Within ice cores many different physical properties are measured, and thus different processes can act to reduce relatively high frequency fluctuations. For example, *Johnsen et al.* [1997] argue that oxygen isotopes diffuse sufficiently rapidly within a core that the annual cycle is greatly attenuated with depth (they give their results by depth rather than by age). However, it is still present throughout the Holocene portion of the record. One must understand this natural filtering for each variable that is sampled before the aliasing hypothesis could be ruled out. Melting multiyear segments of the core to obtain an average (*P. Grootes*, personal communication, 1999) is a very effective filter, but if the segments include fractions of years, there can be considerable leakage nonetheless.

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